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On the Nonlinear Excitation in Self Gravitating Quantum Dusty Plasma

Shalini Bagchi · Kasturi Roy Chowdhury · A.P. Mishra · A. Roy Chowdhury

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Abstract Nonlinear excitation in self gravitating quantum dusty plasma is analysed from the viewpoint of perturbation theory. A plasma configuration consisting of electron, ion and dust particles are treated by reductive perturbation theory, to deduce two coupled modified KdV equations. The soliton like solutions of such coupled systems are obtained with the help of another multiple scale perturbation theory. This leads to the variation of soliton amplitude and velocity due to the several interaction terms.

Keywords Soliton · Jeans frequency · Dusty plasma · Kdv equation · Perturbation theory

1 Introduction

Traditional plasma physics has mainly focussed on regimes characterised by high temperature and low densities, for which quantum mechanical effects have virtually no impact [2]. However recent technological advances have made it mandatory to review the situation. Two of the most important domain of plasma i.e. fusion [11] and space plasma [4] are totally dominated by classical plasma events. On the other hand, it has been observed that quantum effects play a significant role in ultrasmall electronic devices [9], dense astrophysical plasmas [5], Laser plasma [1, 8]. Theoretical studies have already been undertaken to introduce quantum corrections to the Bernstein-Greene-Kruskal [6] equilibria as well as to the Zakharov equation [7]. Quantum magnetohydrodynamics QHD was used by Haas et al. [3]

S. Bagchi
Department of Physics, Jadavpur University, Calcutta 700 032, India

S. Bagchi · K. Roy Chowdhury · A. Roy Chowdhury (✉)
High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta 700 032, India
e-mail: asesh_r@yahoo.com

K. Roy Chowdhury
e-mail: roychowdhury.kasturi@gmail.com

A.P. Mishra
Department of Mathematics, Viswa Varathi University, Santinekaton, West Bengal, India

to establish the equilibrium condition and to analyse the importance of the magnetic field. In the same framework Shukla and Ali [12] obtained a new dispersion relation for electromagnetic drift mode in a non-uniform cold plasma. These types of new dispersion relations show that the quantum corrections are actually of the order of de Broglie wavelength. In the present communication, we have considered a self gravitating quantum dusty plasma and have analysed both the linear and nonlinear regime using a suitable reductive perturbation technique. While the linear regime gives rise to new dispersion relation which gives detailed information about various propagation modes, in the nonlinear part we have obtained two coupled modified KdV equations [10]. These equations are then solved with the hwlp of perturbation technique [13] to obtain the change of amplitude, velocity etc. of the solitary wave.

2 Formulation

The plasma under consideration consists of electron,ion and dust grains with mutual gravitational interaction. Let n_j $(j = i, d, e)$, u_j $(j = i, d, e)$ represent respectively the densities and velocities corresponding to ion, dust and electrons; ϕ, ψ denote the electrostatic and gravitational potentials. Then the equations governing the plasma which takes into account the quantum mechanical corrections can be written as:

$$z_{d_0}\frac{m_e}{m_d}\left(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}\right)=\frac{\partial\phi}{\partial x}-\sigma n_e\frac{\partial n_e}{\partial x}+\frac{H_e^2}{2}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2}{\partial x^2}(\sqrt{n_e})}{\sqrt{n_e}}\right)\tag{1a}$$

$$z_{d_0}\frac{m_i}{m_d}\left(\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}\right)=-Z_i\frac{\partial\phi}{\partial x}-n_i\frac{\partial n_i}{\partial x}+\frac{H_i^2}{2}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2}{\partial x^2}(\sqrt{n_i})}{\sqrt{n_i}}\right)\tag{1b}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}=-\frac{\partial\phi}{\partial x}-\delta n_d\frac{\partial n_d}{\partial x}+\frac{H_d^2}{2}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2}{\partial x^2}(\sqrt{n_d})}{\sqrt{n_d}}\right)-\frac{\partial\psi}{\partial x}\tag{1c}$$

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0\tag{1d}$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e u_e)=0\tag{1e}$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0\tag{1f}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_d+\mu_e n_e-\mu_i n_i\tag{1g}$$

$$\frac{\partial^2\psi}{\partial x^2}=\omega_{jd}^2 n_d+\zeta\omega_{ji}^2 n_i\tag{1h}$$

We now adopt the reductive perturbation technique and introduce the stretched variables

$$\begin{aligned}\zeta &= \epsilon^{\frac{1}{2}}(x-v_0 t)\\ \tau &= \epsilon^{\frac{3}{2}}t\end{aligned}\tag{2}$$

v_0 being the phase velocity. All the dependent variables are also expanded in a series of ϵ;

$$\begin{pmatrix} n_\alpha \\ u_\alpha \\ \phi \\ \psi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \varepsilon \begin{pmatrix} n_{\alpha_1} \\ u_{\alpha_1} \\ \phi_1 \\ \psi_1 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n_{\alpha_2} \\ u_{\alpha_2} \\ \phi_2 \\ \psi_2 \end{pmatrix} \tag{3}$$

Now since (2) implies

$$\frac{\partial}{\partial x} = \epsilon^{\frac{1}{2}} \frac{\partial}{\partial \xi}, \qquad \frac{\partial}{\partial t} = -\epsilon^{\frac{1}{2}} v_0 \frac{\partial}{\partial \xi} + \epsilon^{\frac{3}{2}} \frac{\partial}{\partial \tau}$$

and using (3) in (1a) to (1h) we equate various powers of ϵ.

Thus in the lowest order we get

$$\begin{aligned} n_{e1} &= \frac{m_d \phi_1}{\sigma m_d - Z_{d_0} m_e v_0^2} \\ u_{e1} &= \frac{v_0 \phi_1 m_d}{\sigma m_d - Z_{d_0} m_e v_0^2} \\ n_{i1} &= \frac{m_d (Z_i \phi_1 + \psi_1)}{Z_{d0} m_i v_0^2 - m_d} \\ u_{i1} &= \frac{v_0 m_d (Z_i \phi_1 + \psi_1)}{Z_{d0} m_i v_0^2 - m_d} \end{aligned} \tag{4}$$

along with

$$\begin{aligned} n_{d1} + \mu_e n_{e1} &= \mu_i n_{i1} \\ \omega_{jd}^2 n_{d1} + \zeta \omega_{ji}^2 n_{i1} &= 0 \end{aligned}$$

which finally leads us to the linear dispersion relation;

$$\det \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} = 0 \tag{5}$$

with

$$\begin{aligned} \alpha_{11} &= \frac{1}{v_0^2 - \delta} + \frac{\mu_e m_d}{\sigma m_d - Z_{d_0} m_e v_0^2} - \frac{\mu_i m_d Z_i}{Z_{d0} m_i v_0^2 - m_d} \\ \alpha_{12} &= \frac{1}{v_0^2 - \delta} - \frac{\mu_i m_d}{Z_{d0} m_i v_0^2 - m_d} \\ \alpha_{21} &= \frac{\omega_{jd}^2}{v_0^2 - \delta} + \frac{\zeta \omega_{ji}^2 m_d Z_i}{Z_{d0} m_i v_0^2 - m_d} \\ \alpha_{22} &= \frac{\omega_{jd}^2}{v_0^2 - \delta} + \frac{\zeta \omega_{ji}^2 m_d}{Z_{d0} m_i v_0^2 - m_d} \end{aligned}$$

In the above equations ω_{ji}, ω_{jd} are the corresponding Jeans frequencies given as

$$\omega_{ji} = (4\pi G m_i n_{i0})^{1/2}, \qquad \omega_{jd} = (4\pi G m_d n_{d0})^{1/2} \tag{6}$$

It is important to note that the quantum mechanical terms in (1a) and (1c) has no effect in the dispersion relation.

Now proceeding to higher power of ϵ and after considerable algebraic manipulation we get:

$$\frac{\partial n_{d2}}{\partial \zeta} = \frac{1}{\delta - v_0^2}\left[-\frac{\partial \psi_2}{\partial \zeta} - \frac{\partial \phi_2}{\partial \zeta} - \delta n_{d1}\frac{\partial n_{d1}}{\partial \zeta} + \frac{H_d^2}{4}\frac{\partial^3 n_{d1}}{\partial \zeta^3} - v_0\frac{\partial}{\partial \zeta}(n_{d1}u_{d1}) - v_0\frac{\partial n_{d1}}{\partial \tau} - \frac{\partial v_{d1}}{\partial \tau} - u_{d1}\frac{\partial u_{d1}}{\partial \zeta}\right] \tag{7}$$

$$\frac{\partial n_{e2}}{\partial \zeta} = \frac{1}{\sigma}\left[\frac{\partial \phi_2}{\partial \zeta} - \sigma n_{e1}\frac{\partial n_{e1}}{\partial \zeta} + \frac{H_e^2}{4}\frac{\partial^3 n_{e1}}{\partial \zeta^3} - Z_{d0}\frac{m_e}{m_d}\left(-v_0\frac{\partial u_{e2}}{\partial \zeta} + \frac{\partial u_{e1}}{\partial \tau} + u_{e1}\frac{\partial u_{e1}}{\partial \zeta}\right)\right] \tag{8}$$

$$\frac{\partial n_{i2}}{\partial \zeta} = -Z_i\frac{\partial \phi_2}{\partial \zeta} - n_{i1}\frac{\partial n_{i1}}{\partial \zeta} + \frac{H_i^2}{4}\frac{\partial^3 n_{i1}}{\partial \zeta^3} - \frac{\partial \psi_2}{\partial \zeta} - Z_{d0}\frac{m_i}{m_d}\left(-v_0\frac{\partial u_{i2}}{\partial \zeta} + \frac{\partial u_{i1}}{\partial \tau} + u_{i1}\frac{\partial u_{i1}}{\partial \zeta}\right) \tag{9}$$

The Poisson's equations lead to

$$\frac{\partial^2 \phi_1}{\partial \zeta^2} = n_{d2} + \mu_e n_{e2} - \mu_i n_{i2}$$

$$\frac{\partial^2 \psi_1}{\partial \zeta^2} = \omega_{jd}^2 n_{d2} + \zeta\omega_{ji}^2 n_{i2} \tag{10}$$

Eliminating the higher order terms from (7) to (10) and using the dispersion relation we get two coupled modified KdV equations:

$$A_1\frac{\partial \phi_1}{\partial \tau} + B_1\frac{\partial \psi_1}{\partial \tau} + C_1\phi_1\frac{\partial \phi_1}{\partial \zeta} + D_1\psi_1\frac{\partial \psi_1}{\partial \zeta} + G_1\frac{\partial^3 \phi_1}{\partial \zeta^3} + J_1\frac{\partial^3 \psi_1}{\partial \zeta^3} + K_1\frac{\partial}{\partial \zeta}(\phi_1\psi_1) = 0 \tag{11}$$

$$A_2\frac{\partial \phi_1}{\partial \tau} + B_2\frac{\partial \psi_1}{\partial \tau} + C_2\phi_1\frac{\partial \phi_1}{\partial \zeta} + D_2\psi_1\frac{\partial \psi_1}{\partial \zeta} + G_2\frac{\partial^3 \phi_1}{\partial \zeta^3} + J_2\frac{\partial^3 \psi_1}{\partial \zeta^3} + K_2\frac{\partial}{\partial \zeta}(\phi_1\psi_1) = 0 \tag{12}$$

where the detailed form of the coefficients A_i, B_i etc. $(i = 1, 2)$ are given in the Appendix. The most important point to note is that some of these coefficients do depend on the quantum factor H_i^2 etc. So it can be asserted that the solitary waves do get contribution from the quantum mechanical effects. However, since these are not pure KdV equations, we will have to take recourse to some kind of perturbation theory to estimate in the solitary waveform.

3 Perturbation Technique

To proceed with the solution of the coupled system (11) and (12) we rewrite them as:

$$\frac{\partial \phi_1}{\partial \tau} + P\phi_1\frac{\partial \phi_1}{\partial \zeta} + R\frac{\partial^3 \phi_1}{\partial \zeta^3} = \epsilon M \tag{13}$$

$$\frac{\partial \psi_1}{\partial \tau} + \tilde{P}\psi_1 \frac{\partial \psi_1}{\partial \zeta} + \tilde{R}\frac{\partial^3 \psi_1}{\partial \zeta^3} = \epsilon N \tag{14}$$

where the right hand side contains all the terms that are not included in the standard KdV form. It is well known that if the right hand side is considered to be zero then the solitary wave solution of (13) and (14) can be written as

$$\phi_1 = \alpha \cosh^{-2}(\beta\tau - \Delta\zeta) \tag{15}$$

where $\beta = 4\Delta^3 R$, $\Delta^2 = \frac{P\alpha}{12R}$.

The direct perturbation procedure that we have adopted was originally proposed by Yan and Tang and has been successfully adopted in various situations. It mainly depends on the method of derivative expansion and separation of variable for the linearized problem. An important aspect of this method is that it does not depend on the inverse scattering problem. Using multiple scales we introduce the independent time variable

$$t_n = \epsilon^n t, \quad n = 0, 1, 2, \dots \tag{16}$$

at the same time

$$\delta_t = \delta_{t0} + \epsilon\delta_{t1} + \epsilon^2\delta_{t2} \tag{17}$$

whereas the nonlinear field variables ϕ, ψ are expanded as

$$\phi = \phi^0 + \epsilon\phi^1 + \epsilon^2\phi^2 + \cdots \tag{18}$$

$$\psi = \psi^0 + \epsilon\psi^1 + \epsilon^2\psi^2 + \cdots \tag{19}$$

Substituting in (13) and also in (14) we get the following sets of equations; where M^0, $4M^1$, M^2 are the expansion coefficients on the right hand side of (13) and (14).

The first three equations obtained by us are

$$\frac{\partial \phi_1^0}{\partial \tau_0} + P\phi_1^0\frac{\partial \phi_1^0}{\partial \zeta} + R\frac{\partial^3 \phi_1^0}{\partial \zeta^3} = 0 \tag{20}$$

$$\frac{\partial \phi_1^0}{\partial \tau_1} + \frac{\partial \phi_1^1}{\partial \tau_0} + P\left(\phi_1^0\frac{\partial \phi_1^1}{\partial \zeta} + \phi_1^1\frac{\partial \phi_1^0}{\partial \zeta}\right) + R\frac{\partial^3 \phi_1^{(1)}}{\partial \zeta^3} = M^{(1)} \tag{21}$$

$$\frac{\partial \phi_1^2}{\partial \tau_0} + \frac{\partial \phi_1^1}{\partial \tau_1} + \frac{\partial \phi_1^0}{\partial \tau_2} + P\left(\phi_1^0\frac{\partial \phi_1^2}{\partial \zeta} + \phi_1^1\frac{\partial \phi_1^1}{\partial \zeta}\right) + \phi_1^2\frac{\partial \phi_1^0}{\partial \zeta} + R\frac{\partial^3 \phi_1^2}{\partial \zeta^3} = M^{(2)} \tag{22}$$

The initial condition of the system is taken as;

$$\begin{aligned} &\phi_1^0(\xi, 0) = \alpha\cosh^{-2}(-\sigma\xi) \\ &\phi_1^n(\xi, 0) = 0, \quad n = 1, 2, 3, \dots \end{aligned} \tag{23}$$

One should note that the zeroeth order (20) is just the KdV equation. Its single soliton solution may be written as

$$\begin{aligned} \phi_1^0(\xi, \tau_0) &= \alpha\cosh^{-2} z \\ Z &= -\sigma(\xi - V), \qquad V_{\tau_0} = \frac{\beta}{\sigma} \end{aligned} \tag{24}$$

The basic approach to the present perturbation methodology is to consider the soliton parameters α, σ and V to be functions of slow time variables introduced as in (16), τ_1, τ_2 etc.; but α is independent of τ_0 and that of V is given as

$$V_{\tau_0} = \frac{\beta}{\sigma} \tag{25}$$

It follows from (24)

$$\frac{\partial \phi_1^0}{\partial \tau_n} = \alpha_{\tau n} U_1(z) - 2\alpha\sigma V_{\tau n} U_2(z) \tag{26}$$

for $n = 1, 2, 3, \ldots$ where $U_1(z)$ and $U_2(z)$ are given as;

$$\begin{aligned} U_1(z) &= (1 - z \tanh z) \cosh^{-2}(z) \\ U_2(z) &= \cosh^{-2} z \tanh z \end{aligned} \tag{27}$$

Then the linearized KdV equations (21) and (22) together with the appropriate initial conditions (23) are reduced to the following compact form:

$$\phi_{1\tau_0}^1 - R\sigma^3 \hat{L}\phi_1^1 = F^{(1)}(z) = M^{(1)} - \alpha_{\tau_1} U_1(z) + 2\alpha\sigma V_{\tau_1} U_2(z) \tag{28a}$$

$$\phi_1^1(z, 0) = 0 \tag{28b}$$

$$\phi_{1\tau_0}^2 - R\sigma^3 \hat{L}\phi_1^2 = F^{(2)}(z) = M^{(2)} - \alpha_{\tau_2} U_1(z) + 2\alpha\sigma V_{\tau_2} U_2(z) - \phi_{1\tau_1}^1 + P\sigma\phi_1^{(1)}\phi_{1z}^{(1)} \tag{29a}$$

$$\phi_1^2(z, 0) = 0 \tag{29b}$$

where z is the space coordinate in a system moving with the soliton and $\hat{L}$ stands for

$$\hat{L} = \frac{d^3}{dz^3} + (12\cosh^{-2} z - 4)\frac{d}{dz} - 24\cosh^{-2} z \tanh z \tag{30}$$

whose adjoint is

$$\hat{L}^+ = \frac{d^3}{dz^3} + (12\cosh^{-2} z - 4)\frac{d}{dz} \tag{31}$$

In the next section we show how to solve (28) and (29) which are nothing but the linearized KdV equations.

4 Linearized Equation and Eigenvalue Problem

The equations (28) and (29) are usually solved with the help of separation of variables, for which the key lies in the analysis of the eigenvalue problem of operators $\hat{L}$ and its adjoint $\hat{L}^+$

$$\hat{L}u = \lambda u \tag{32a}$$

$$\hat{L}^+ v = \lambda^1 v \tag{32b}$$

The eigenfunction of (32a) for continuous eigenvalues

$$\lambda = -ik(k^2+4) \tag{33}$$

is given as

$$u(z,k) = \frac{1}{\sqrt{2\pi}k(k^2+4)}[k(k^2+4) + 4i(k^2+2)\tanh z - 8k\tanh^2 z - 8i\tanh^3 z]\exp ikz \tag{34}$$

Also for $\lambda = 0$, we get

$$u_2(z) = \tanh z \cosh^{-2} z$$

Similarly for the adjoint (32b) we have

$$v(z,k) = \frac{1}{\sqrt{2\pi}k(k^2+4)}[k^2 - 4ik\tanh z - 4\tanh^2 z]\exp -ikz \tag{35}$$

for

$$\lambda^1 = ik(k^2+4)$$

and

$$v_2(z) = \cosh^{-2} z \quad \mathit{for}\ \lambda^1 = 0 \tag{36}$$

But for completeness one needs to include

$$u_1(z) = (1 - z\tanh z)\cosh^{-2} z \tag{37}$$

which satisfies;

$$\hat{L}u_1(z) = -8v_2(z) \tag{38}$$

Similarly for the adjoint problem.

The completeness relation is written as, for $\{u\} = \{v(z,k), u_j(z), j = 1, 2\}$ and $\{v\} = \{v(z,k), v_j(z), j = 1, 2\}$

$$P_2\int_{-\infty}^{\infty} u(z,k)v(z',k)dk + \sum_{j=1}^{2} u_j(z)v_j(z') = \delta(z-z') \tag{39}$$

where P_2 stands for Principal value of the integral. We also use the relations

$$\int_{-\infty}^{\infty} u(z,k)v(z,k')dz = \delta(k-k') \tag{40}$$

and the orthogonality relations

$$\begin{aligned} \int_{-\infty}^{\infty} u(z,k)v_j(z)dz &= \int_{-\infty}^{\infty} v(z,k)u_j(z)dz = 0 \\ \int_{-\infty}^{\infty} u_j(z)v_k(z)dz &= \delta_{jk} \end{aligned} \tag{41}$$

Based on this completeness theory any function $F(z)$ can be expanded as;

$$F(z) = P_2 \int_{-\infty}^{\infty} f(k)u(z,k)dk + \Sigma f_j u_j(z) \tag{42}$$

where:

$$\begin{aligned} f(k) &= \int_{-\infty}^{\infty} F(k)v(z,k)dz \\ f_j &= \int_{-\infty}^{\infty} F(z)v_j(z)dz \end{aligned} \tag{43}$$

5 Effects of Perturbation on Soliton:

We now apply perturbation technique to ϕ_1^1 and $F^{(1)}$ using the basis function $\{u\}$. For example ϕ_1^1 can be written as

$$\phi_1^1(z,\tau_0) = P_2 \int T^{(1)}(\tau_0,k)u(z,k)dk + \sum T_j^{(1)}(\tau_0)u_j z \tag{44}$$

$$\begin{aligned} F^{(1)} &= M^{(1)} - \alpha_{\tau_1} v_1(z) + 2\alpha\sigma V_{\tau_1} u_2(z) \\ &= P_2 \int f^{(1)}(k)u(z,k)dk + \Sigma f_j^{(1)} u_j(z) \end{aligned} \tag{45}$$

The expansion coefficients are obtained via orthogonality relations. Substituting (44) and (45) in (20), (21), (22) and using $\hat{L}u(z,k) = -ik(k^2+4)u(z,k)$ and $\hat{L}u_2(z) = 0$ we get

$$\begin{aligned} \frac{\partial T_0^{(1)}(\tau_0,k)}{\partial \tau_0} + ik(k^2+4)R\sigma^3 T^{(1)}(\tau_0,k) &= f^{(1)}(k) \\ T^{(1)}(0,k) &= 0 \\ \frac{\partial T_1^{(1)}(\tau_0)}{\partial \tau_0} &= f_1^{(1)} \\ T_1^{(1)}(0) &= 0 \\ \frac{\partial T_2^{(1)}(\tau_0,k)}{\partial \tau_0} + 8R\sigma^3 T^{(1)}(\tau_0) &= f_2^{(1)}(k) \\ T_2^{(1)}(0) &= 0 \end{aligned}$$

Integration of these equations lead to

$$\begin{aligned} \alpha_{\tau_1} &= \int [M^{(1)} \cosh^{-2} z]dz \\ V_{\tau_1} &= -\frac{1}{2\alpha\sigma} \int [M^{(1)}(\tanh z + z\cosh^{-2} z)]dz \end{aligned}$$

which gives the variation of α and V due to the perturbation $M^{(1)}$. Similarly we can find that $\phi_1^1(z,\tau_0)$ is given as

$$\phi_1^1(z,\tau_0) = P_2 \int_{-\infty}^{\infty} \left[\frac{f^{(1)}(k)}{ik(k^2+4)R\sigma^3} \{1-\exp[-ik(k^2+4)R\sigma^3\tau_0]\} u(z,k) \right] dk$$

In a similar manner all higher order perturbation effects may be calculated.

6 Conclusion

In our above analysis we have considered the formation of nonlinear excitations in a self gravitating quantum dusty plasma through reductive perturbation technique. Two coupled modified KdV equations are derived. The solutions to this system are obtained via another multiple scale perturbation theory based on the completeness of eigenfunctions around the soliton solution. A detailed numerical analysis of such a system will be taken up in a future communication.

Appendix

$$A_1 = \frac{2v_0}{(v_0^2-\delta)^2} - \frac{2\lambda_1\mu_e v_0}{(\sigma-\lambda_1 v_0^2)^2} - \frac{\lambda_2\mu_i v_0 Z_i}{(1-\lambda_2 v_0^2)^2}$$

$$B_1 = \frac{2v_0}{(v_0^2-\delta)^2} - \frac{2\lambda_2\mu_i v_0}{(1-\lambda_2 v_0^2)^2}$$

$$C_1 = \frac{3v_0^2+\delta}{(v_0^2-\delta)^3} - \frac{\mu e(\sigma+3\lambda_1 v_0^2)}{(\sigma-\lambda_1 v_0^2)^3}$$

$$D_1 = \frac{3v_0^2+\delta}{(v_0^2-\delta)^3} + \frac{\mu i(1+3\lambda_2 v_0^2)}{(1-\lambda_2 v_0^2)^3}$$

$$G_1 = \frac{\mu e H_e^2}{4(\sigma-\lambda_1 v_0^2)^2} + \frac{\mu i H_i^2 Z_i}{4(1-\lambda_2 v_0^2)^2} - \frac{H_d^2}{4(v_0^2-\delta)^2} - 1$$

$$H_1 = \frac{\mu i H_i^2}{4(1-\lambda_2 v_0^2)^2} - \frac{H_d^2}{4(v_0^2-\delta)^2}$$

$$K_1 = \frac{3v_0^2+\delta}{(v_0^2-\delta)^3} + \frac{\mu i Zi(1+3\lambda_2 v_0^2)}{(1-\lambda_2 v_0^2)^3}$$

$$A_2 = \frac{2v_0\omega_{jd}^2}{(v_0^2-\delta)^2} + \frac{2\zeta\omega_{ji}^2\lambda_2 v_0 Z_i}{(1-\lambda_2 v_0^2)^2}$$

$$B_2 = \frac{2v_0\omega_{jd}^2}{(v_0^2-\delta)^2} + \frac{2\zeta\omega_{ji}^2\lambda_2 v_0}{(1-\lambda_2 v_0^2)^2}$$

$$C_2 = \frac{\omega_{jd}^2(3v_0^2+\delta)}{(v_0^2-\delta)^3} - \frac{\zeta\omega_{ji}^2 Z_i^2(1+3\lambda_2 v_0^2)}{(1-\lambda_2 v_0^2)^3}$$

$$D_2 = \frac{\omega_{jd}^2(3v_0^2+\delta)}{(v_0^2-\delta)^3} - \frac{\zeta\omega_{ji}^2(1+3\lambda_2 v_0^2)}{(1-\lambda_2 v_0^2)^3}$$

$$G_2 = \frac{\omega_{jd}^2 H_d^2}{4(v_0^2-\delta)^2} - \frac{\zeta\omega_{ji}^2 Z_i H_i^2}{4(1-\lambda_2 v_0^2)^2}$$

$$H_2 = \frac{\omega_{jd}^2 H_d^2}{4(v_0^2-\delta)^2} - \frac{\zeta\omega_{ji}^2 H_i^2}{4(1-\lambda_2 v_0^2)^2} - 1$$

$$K_2 = \frac{\omega_{jd}^2(3v_0^2+\delta)}{(v_0^2-\delta)^3} - \frac{\zeta\omega_{ji}^2 Z_i(1+3\lambda_2 v_0^2)}{(1-\lambda_2 v_0^2)^3}$$

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